* **Probability Density Function (PDF) of a RV**
* **Cumulative Density Function (CDF) of a RV**
* **Joint PDF of**

If and are independent RVs, then

Let and and are independent, then PDF of is given as

* **Expectation (or called ‘Mean’ or ‘Average’)**

Expectation of a RV is denoted by or or

For another RV which is a function of , i.e.,

* **Variance**

Variance of a RV is denoted by or

* **Scaling, constant addition, and sum of RVs**

If , where is a RV, and are constants, then

If , where and are RVs, and are constants, then

Where and denotes the joint PDF of . If and independent, .

So,

Finally,

* **Weighted sum of RVs**

If is a weighted sum of RVs, i.e. where are RVs and are their weights, then

if are independent,

If are **Independent and Identically Distributed (IID)** RVs and we consider their sample mean then, this is a special case of weighted sum, i.e., all the weights are equal to and

* **PDF of a Uniform RV**
* **PDF of a Gaussian RV**
* **Rayleigh RV**

Consider a complex Gaussian RV , i.e.,

where and and they are independent.

Let denote the magnitude of , i.e.,

Then, has a Rayleigh distribution given as

* **Q-function**

Let be a Gaussian RV such that , then the defined as the probability of being larger than is given as

Let be a Gaussian RV with general mean and variance, i.e., , then probability of being larger than is given as

Replacing by

The Q-function can be expressed in terms of the error function, as

* **Gaussian RV**

Linear operation of Gaussian RV(s) results in another Gaussian RV.

If where are independent Gaussian RVs, then, is the Gaussian RV given as

* **Central Limit Theory (CLT)**

Consider a summation of RVs, i.e.,

According to CLT, as increases, the PDF of converges to Gaussian distribution.

Assume the case when are independent, then

* **Random Process**

Each waveform(signal) is corresponds to the outcome(observation) of . So, is the -th outcome of

* **Ensemble mean of Random Process**

where denotes the PDF of at

* **Ergodic Random Process**

There exist lots of ensemble mean for the different statics, i.e.

If all these ensemble means are equal to their time average versions, i.e.

then, the Random Process is said to be *ergodic*

* **Autocorrelation of a random process**

where denotes ensemble mean

* **Wide Sense Stationary (WSS)**

If the followings are satisfied, is said to be *Wide Sense Stationary*.

1. constant irrespective of
2. is determined only by not individually by or , i.e., is simply expressed as where .

WSS process is not always ergodic, but all ergodic processes are WSS. So, ergodic process is the subset of WSS process.

* **Property of Autocorrelation for WSS process**
  + of the process if ergodic.
  + magnitude: even fn., phase: odd fn.  
     for real-valued processes.
* **Power Spectral Density (PSD)**

Fourier transform of also randomly changes because is a random process. So, we can’t use Fourier transform of for analyzing Fourier transform of in the frequency domain.

However, we can calculate PSD of as follows

Note that is **deterministic** but describes the spectrum of in frequency domain.

* **Property of PSD**
  + Assume that a process passes through a linear system. Then, the PSD of output process is given by

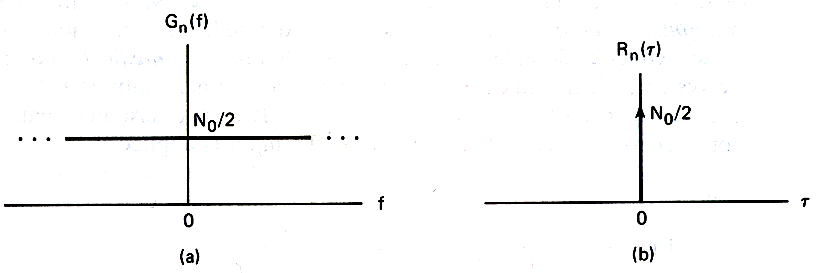
where of the linear system.

* **Gaussian Random Process**

A random process is a Gaussian random process if for all and all time instances , the random variable set have a jointly Gaussian PDF.

* **Theorem of Gaussian Random Process**
  + Any sample point from a Gaussian random process, i.e., for any time instance has a Gaussian PDF.
  + For Gaussian processes, knowledge of the mean and autocorrelation fives a complete statistical description of the process.
  + If the Gaussian process is passed through a linear system, then the output process will be another Gaussian process.
* **White Process**

A Process is called a white process if it has a flat PSD, i.e., is a constant for all .



* **Additive White Gaussian Noise (AWGN)**

The term AWGN self-explains the characteristics of the background noise cause the **background noise** is Additive, White and Gaussian process

* **Decision Variable**

Sample at to generate the decision variable

Signal component , Noise component is a Gaussian . So,

* **Maximum Likelihood Detection**

is called ‘Likelihood ratio’

* **Probability of ML Detection Error**



자세한 건 LAB16 참고

* **Inner Product of Two Waveforms and**

over , is

* **Orthogonal Signal Set**

A set of waveforms, is said to form an orthogonal set over ,

is the Energy of -th waveform. and with are said to be orthogonal each other.

* **Orthonormal Set**

If of Orthogonal Set, then the set is said to be and orthonormal set.

* **Signal Generation using Orthonormal Set**
* **Signal Vector Space**

The waveform is mapped into a vector in the -dimensional space spanned by

* **Find Axis Components**
* **Equivalence Properties**

|  |  |  |
| --- | --- | --- |
|  | Waveform Space | Vector Space |
| Signal representation |  |  |
| Energy |  |  |
| Difference Energy |  |  |
| Correlation (Inner product) |  |  |
| Orthogonality |  |  |

* **Proof of**
* **Proof of**
* **ML Detection in Vector Space**
* **AWGN in Signal Vector Space**

AWGN signal is mapped into a vector in the signal vector space.

where

* + are Gaussian RVs.
  + Specifically,
  + are i.i.d.
* **Proof of**
* **Proof of** independent

Independent if Gaussian and no correlation.

* **Correlator-based ML detection**

if all -ary symbols have identical energy, i.e.,

* **Antipodal signaling**
* ML Detection of Binary Signals

if

* Bit Error Rate (BER) of MLD of Binary Signal